Initial Study on Expendable Acoustic Countermeasures for Torpedo Defence

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Abstract
An initial study into the effectiveness of expendable acoustic countermeasures on passive acoustic homing torpedoes has been conducted. The scenario has been kept generic to enable open discussion of results. Future evolutions of the study will consider more realistic, and possibly classified, torpedo and decoy models. The results indicate that expendable countermeasures can be effective against passive homing torpedoes if the deployment range from the ship is selected appropriately. Surprisingly, early decoy deployment may not be necessary to ensure ship survival. Deployment pattern was more important in determining scenario outcome than deployment time.
EXECUTIVE SUMMARY

BACKGROUND
Acoustic homing torpedoes are increasingly capable and the traditional shipborn countermeasure - the towed acoustic decoy - may no longer be effective against the latest generation of weapons. Alternatively, expendable acoustic decoys are available which can be deployed by rocket, pneumatic launcher, or by hand. The effectiveness of expendable decoys will depend on the number and pattern of deployment.

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AIM
To simulate the effectiveness of expendable acoustic countermeasures on passive homing torpedoes.

RESULTS
The results of the initial study indicate that expendable acoustic countermeasures can be effective against passive homing torpedoes if the deployment range from the ship is selected appropriately. Early decoy deployment may not be necessary to ensure ship survival. Deployment pattern was more important in determining scenario outcome than deployment time.
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1 Introduction

Torpedoes are a rare but potent threat to shipping. The most recent torpedo attack occurred in March 2010 when the ROKS Cheonan, a South Korean corvette, split in two after an explosion and sank. The official report into the incident claimed the sinking was caused by a CHT-02D torpedo fired by a North Korean submarine [1].

Anti-shipping torpedoes may use active and passive sonar to detect and home on a target vessel. These types of weapon are known as acoustic homing torpedoes. There is also a class of torpedo that homes indirectly on the target vessel’s wake, such a weapon is known as a wake homing torpedo (WHT).

Options for countering acoustic homing torpedoes currently include evasive manoeuvre, and towed and expendable acoustic countermeasures. Towed countermeasures, for example the AN/SLQ-25A towed decoy, are acoustic noise sources which are towed underwater behind the vessel to be protected. They are usually louder than the surface vessel and are intended to cause the torpedo to repeatedly home on the decoy rather than the vessel. Expendable countermeasures, by contrast, are self-contained acoustic sources which are launched from the ship and begin operation shortly after splashdown at sea. They continue to transmit until their energy supply is exhausted.

The development of countermeasures aimed at protecting ships against specific torpedo threats is a challenging problem. Ideally, one would conduct sea trials to test candidate countermeasures against real weapons. In practice, it is not possible to thoroughly test all countermeasure permutations in sea trials. Some form of pre-screening is needed to evaluate promising solutions which can then be verified, in limited configurations, in sea trials.

The use of computer simulation to assess countermeasure effectiveness is the only alternative to full-scale sea trials, and this technique is used by a number of different nations. Canada, Norway and the UK use the Odin model originally developed by DERA (and subsequently QinetiQ) in the UK. Both the USA and Australia use the Technology Requirements Model (TRM) developed at Pennsylvania State University. New Zealand now uses the Torpedoes Against Surface Targets (TOAST) model first developed at DTA in 1999.

1.1 The AG-3 engagement model comparison study

Between 2002 and 2006 collaborative work under TTCP in torpedo defence took place in MAR AG-3. At the AG-3 meeting in October 2003 it was decided to conduct a comparison of Odin, TRM and TOAST on a torpedo attack scenario involving a passive acoustic homing
torpedo against a frigate size surface vessel protected by a towed acoustic countermeasure. Certain aspects of the scenario were made generic in order to permit open discussion of the simulation results, and to circulate the study definition to all nations. The purpose of the work was to cross-validate the participating models on a common scenario. The study began in 2005, and final results were reported in 2007. A good level of agreement was obtained between the models.

1.2 The KTA-7 expendable acoustic decoys study

Since 2007, collaborative work in torpedo defence has continued as KTA-7 under MAR TP-9. At the TP-9 annual meeting in September 2010 it was proposed that a series of engagement model studies be conducted focusing on practical problems, rather than model validation. The purpose of the collaboration was twofold:

- improving modeling expertise in the participating nations;
- producing results that assist tactics development.

The first area proposed for KTA-7 collaboration was an engagement model scenario to explore the effectiveness of expendable acoustic decoys against passive acoustic homing torpedoes. The study could be extended by considering the effectiveness of expendable decoys in conjunction with a towed acoustic countermeasure, and against both passive and active-passive homing torpedoes. For simplicity, the initial effort considered only iso-velocity sound speed profiles. Future evolutions of the scenario may be developed to assess the effects of different environmental conditions.

The engagement model scenario was constructed using previously developed ship and torpedo models and using broadband deployed acoustic sources to model the decoys. Variations in decoy deployment distance from the ship have been considered and the optimal distance determined. The scenario was kept deliberately unclassified to facilitate the sharing of models and results. It is envisaged that subsequent work will include more realistic (and possibly classified) torpedo, ship and countermeasure models.

1.3 Report outline

New Zealand has completed its work on this scenario using the TOAST engagement model. This report describes the scenario and its component models in detail, and presents results and conclusions from the engagement model runs.

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4Key Task Area on Surface Ship Torpedo Defence.
5Technical Panel 9 on Anti-Submarine Warfare Systems and Technology.
An overview of the engagement model scenario is first presented in section 2. Details of the component models are given in subsequent sections. The acoustic environment used is described in section 3, the torpedo model is presented in section 4, and the ship model in section 5.

Three different versions of the scenario were modeled (labeled A, B and C). Scenario A is the baseline scenario with the initially agreed upon torpedo, ship and countermeasure models. Results for Scenario A are given in section 6. In Scenario B the effect of reduced torpedo warning time on the survival probability of the ship was explored, results are given in section 7. Scenario C explores the effect of alternative torpedo counter-countermeasures (a snake-step following an encounter with a decoy), results are given in section 8.

A summary and conclusions of the study are given in section 9.

2 Scenario description

The study is intended to explore the effectiveness of expendable decoys in countering heavy-weight passive homing torpedoes. In each case the scenario concerns an engagement between an alerted ship and a single torpedo launched at a random location.

At time $t = 0$ the ship is located at the coordinate system origin and is heading north at 15 knots. The torpedo is launched at $t = 0$ from a distance of $6 \pm 1$ km from the ship, randomly distributed in bearing from starboard $30^\circ$ to $150^\circ$ relative to the ship’s initial heading. The torpedo is launched on an intercept course, as described in 4.3. The torpedo conducts a linear run-out until a time equal to 66% of the calculated intercept time, then begins an acoustic search phase.

The ship has a generic non-specified sonar system which detects the torpedo at $3 \pm 0.5$ km, then begins an evasion manoeuvre consisting of a $135^\circ$ turn to port.

The threat weapon has the characteristics typical of a dual speed electrically propelled weapon, with a search speed of 30 knots and a homing speed of 40 knots. The torpedo runs at a constant depth of 15 m through the scenario. The torpedo sonar operates at 25 kHz with a bandwidth of 2 kHz. It is based on the torpedo sonar array given in Ref. [2] and is described in detail in section 4.4 of this report.

The sound speed profile is assumed to be isovelocity. In later scenario evolutions more realistic sound speed profiles may be used.

Each expendable acoustic countermeasure is launched to a range varying between 10 to 2000 m, at a bearing of $\pm 90^\circ$ relative to the instantaneous ship heading. The decoys are launched singly, commencing at the time of torpedo detection and then at 30, 60, and 90
seconds subsequently, alternating between port and starboard directions (the first decoy is launched to starboard towards the incoming torpedo). Each decoy begins transmitting after a time delay proportional to the launch range (assuming a horizontal component of airspeed of 100 m/s) plus 10 s following splashdown. The decoys transmit at a depth of 15 m at the splashdown position and remain static throughout the rest of the scenario. The acoustic component of the decoy is an omni directional, broadband noise source operating continuously with a lifetime of 8 minutes.

Simulations of the scenario are run 1000 times, with the torpedo launch range and bearing randomized uniformly at the start of each run. For each run the detection time, hit or miss, time-to-hit, and the number of decoy detection events by the torpedo are recorded (that is, the instances where the torpedo recognizes it is homing on a countermeasure rather than the target). From the ensemble of 1000 runs, the hit probability and distributions of detection time, time-to-hit, and number of decoy detection events can be calculated.

3 Environment

In the initial phases of the study an iso-velocity acoustic environment with volume attenuation is assumed. The transmission loss $TL$ is then given by

$$TL = 20 \log_{10} (R + R_{\text{min}}) + \alpha R$$

(1)

where $R$ is the range, $R_{\text{min}}$ is the minimum range and $\alpha$ is the logarithmic absorption coefficient. The assumed frequency dependence of the absorption coefficient is given in table 1. Values of $\alpha$ between the tabulated frequencies may be found by linear interpolation.

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>$\alpha$ (dB/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>5.5</td>
</tr>
<tr>
<td>26</td>
<td>6.3</td>
</tr>
</tbody>
</table>

**Table 1:** The logarithmic absorption coefficient $\alpha$ as a function of frequency.

A significant feature of the standard form of the geometric spreading law (Eq. (1) with $R_{\text{min}} = 0$) is that there is no lower bound on the propagation loss. This is an important consideration because TOAST assumes all acoustic sources and receivers are point-like entities for the purposes of propagation loss calculation. The parameter $R_{\text{min}}$ has been introduced in (1) to limit the received acoustic level when source and receiver make a close approach. This situation may occur in the present scenario since the torpedo and the decoys operate at the
same depth (15 m) and they are treated as point like objects. Hence this limit is needed to avoid the possibility of numerical overflow.

Later scenario evolutions may consider typical winter and summer velocity profiles. Winter profiles are upward refracting and may result in the formation of a surface duct. This reduces transmission loss between sources and receivers that are located in the duct. Summer profiles are downward refracting and result in the formation of shadow zones between sources and receivers near the sea surface. Summer profiles typically increase transmission loss.

4 Torpedo

4.1 Dynamics

The threat torpedo’s dynamics are modeled by a modified version of Odin’s *SimpleMotion* model, which is described below. The Odin *SimpleMotion* model is intended for small vehicles when one is not interested in the subtleties of the motion, or when not much information about the dynamic characteristics of the vehicle is available. In this scenario the torpedo is highly maneuverable compared to the target vessel and there is not a compelling reason to introduce a more detailed dynamic model.

Dynamics models assume a right-handed coordinate system with the $y$ axis oriented north, as illustrated in Fig. 1.

![Figure 1: The fixed coordinate system assumed in this study (the $z$ axis points out of the page, so that the system is right-handed). The body coordinate vector $x'$ is aligned with the longitudinal axis of the body. The angle $\phi$ is the yaw angle of the body with respect to the fixed frame.](image)

Vehicle course in *SimpleMotion* is controlled by the turn rate $R$, speed by the linear accel-
eration $a$ and depth by the pitch rate $P$.

Course changes in the SimpleMotion model are effected by changing the vehicle’s turn rate $R$ according to

$$ R = \begin{cases} 
0, & |\phi_{\text{err}}| < \phi_{\text{tol}}, \\
\text{sign}(\phi_{\text{err}})R_{\text{max}}, & |\phi_{\text{err}}| \geq \phi_{\text{tol}},
\end{cases} \quad (2) $$

where $\phi$ is the present course, $\phi_{\text{dem}}$ is the demanded or ordered course and

$$ \phi_{\text{err}} = \phi_{\text{dem}} - \phi \quad (3) $$

is the course error. The quantity $\phi_{\text{tol}}$ is called the course tolerance, which sets the precision with which the torpedo course may be specified (in real torpedoes there is also random variations in torpedo course, although that aspect of the problem is not modeled here). Values of $\phi_{\text{tol}} \sim 1^\circ$ are suggested. When $|\phi_{\text{err}}| < \phi_{\text{tol}}$ one sets $\phi = \phi_{\text{dem}}$.

The vehicle’s heading can in principle be updated using

$$ \dot{\phi} = R, \quad (4) $$

but direct integration of this equation is problematic when the discontinuity in the course angle is encountered. An approach which avoids this problem is to use the direction cosines $\alpha = \cos \phi$ on the $x$-axis and $\beta = \sin \phi$ on the $y$-axis. The direction cosines satisfy

$$ \frac{d\alpha}{dt} = -\beta R, \quad \frac{d\beta}{dt} = \alpha R \quad (5) $$

when $\dot{\phi} = R$. These equations can be used to update the course angle indirectly, via $\phi = \arctan(\beta, \alpha)$, which avoids the $2\pi$ discontinuity problem.

It is easiest to derive the equation for the course error in the complex plane, using the complex numbers $u = e^{i\phi}$ and $u_{\text{dem}} = e^{i\phi_{\text{dem}}}$. We then have

$$ u_{\text{dem}}u^* = e^{i(\phi_{\text{dem}} - \phi)} = e^{i\phi_{\text{err}}} \quad (6) $$

and hence

$$ \text{Re} \ u_{\text{dem}}u^* = \alpha \alpha_{\text{dem}} + \beta \beta_{\text{dem}}, \quad (7) $$

$$ \text{Im} \ u_{\text{dem}}u^* = \alpha \beta_{\text{dem}} - \beta \alpha_{\text{dem}}. \quad (8) $$

Using the four-quadrant arctangent function the course error in terms of these direction
\[ \phi_{err} = \arctan(\alpha \beta_{dem} - \beta \alpha_{dem}, \alpha \alpha_{dem} + \beta \beta_{dem}) . \]  

(9)

The vessel’s linear acceleration \( a \) is changed in response to a speed demand \( v_{dem} \) as

\[
a = \begin{cases} 
0, & |v - v_{dem}| < v_{tol} , \\ 
a_{\text{max}}, & |v - v_{dem}| \geq v_{tol} , 
\end{cases}
\]

(10)

and one sets \( v = v_{dem} \) when \( |v - v_{dem}| < v_{tol} \).

In Odin’s SimpleMotion model the pitch changes discontinuously in response to depth change demands, as long as the depth change is larger than a certain tolerance. This can result in the pitch oscillating between its maximum and minimum values as the weapon approaches its demanded depth. However, this is not an issue in the present scenario as the torpedo depth is fixed at 15 m.

The threat weapon has characteristics typical of a dual speed electrically propelled weapon, with a search speed of 30 kn and a homing speed of 40 kn. It runs at a depth of 15 metres throughout the scenario. In this study the values \( a_{\text{max}} = 2 \text{ m/s}^2 \) and \( R_{\text{max}} = 10 \text{ deg/s} \) were used, which are similar to the class B generic torpedo [3]. As noted above, the torpedo runs at a constant depth throughout and so no depth change demands are made.

### 4.2 Endurance

It is not necessary in this scenario to model the torpedo’s propulsion system in detail. The torpedo is assumed to be a dual-speed electrically driven weapon, with a search speed of 30 kn and a homing speed of 40 kn. The torpedo’s battery capacity is based on that of the F17P mod 2, which has a maximum range reported to be 30 km at a speed of 35 kn [4]. This level of endurance is 50% greater than that used in the 2007 AG-3 study. It is believed to be more representative of modern electrically propelled heavyweight torpedoes.

Assuming the drag force on the torpedo to be proportional to the square of the speed, the total energy required to propel the weapon at speed \( V \) for a duration \( T \) is proportional to \( V^3T \). This leads to the energy consumption rates given in table 2. Fuel consumption rates in between the tabulated values are determined by linear interpolation in the simulation. The given rates assume the initial stored energy is equal to 1500 (in arbitrary units).
4.3 Fire control solution

The initial course of the torpedo from its starting point is the intercept course, that is, the
course which leads to a collision if both the target ship and torpedo maintain their initial
courses and speeds. Modern torpedoes may use more sophisticated fire control methods.
For example, a torpedo may wish to keep the target vessel within its beam pattern as it
approaches so it can make corrections if the target speed or course varies.

Let the initial position of the torpedo be \((x_0, y_0)\) and its speed be \(v_T\). At \(t = 0\) the target
vessel is assumed to be at the position \((0, 0)\), and is assumed to be travelling on a heading
\(\phi_S\) at speed \(v_S\). The torpedo chooses a heading \(\phi_T\) to collide with the target assuming the
target maintains its initial course and speed, and the weapon maintains speed \(v_T\).

The trajectory of the ship is

\[
(x_S, y_S) = v_S(\cos \phi_S, \sin \phi_S) t,
\]

while the trajectory of the torpedo is

\[
(x_T, y_T) = v_T(\cos \phi_T, \sin \phi_T) t + (x_0, y_0).
\]

Define the perpendicular distance between the point \((x_0, y_0)\) and the ship’s track as \(D_\perp\),
where

\[
D_\perp = x_0 \sin \phi_S - y_0 \cos \phi_S.
\]

Then equating the ship and torpedo trajectories at time \(t\) leads to

\[
x_0 \sin \phi_T - y_0 \cos \phi_T = \frac{v_S}{v_T} D_\perp.
\]
Defining first the quantity

\[ \Delta = \sqrt{x_0^2 + y_0^2 - \left(\frac{v_S}{v_T}\right)^2 D_\perp^2} \]  

(15)

it can be shown that there are two solutions to (14) for \( \phi_T \) which are given by

\[ \sin \phi_T^\pm = \frac{\mp y_0 \Delta + x_0 \frac{v_S}{v_T} D_\perp}{x_0^2 + y_0^2}, \]  

(16a)

\[ \cos \phi_T^\pm = \frac{\mp x_0 \Delta - y_0 \frac{v_S}{v_T} D_\perp}{x_0^2 + y_0^2}. \]  

(16b)

In terms of the four-quadrant arctangent function, \( \arctan(y, x) \), one then has

\[ \phi_T^\pm = \arctan\left(\mp y_0 \Delta + x_0 \frac{v_S}{v_T} D_\perp, \mp x_0 \Delta - y_0 \frac{v_S}{v_T} D_\perp\right). \]  

(17)

There are two solutions to this problem. One solution corresponds to a collision for times \( t > 0 \), the other for \( t < 0 \). We require an intercept at a positive time since that is the physically realizable solution.

The time to intercept, \( t_I \), is given by

\[ t_I = \frac{x_0 + y_0}{v_S (\cos \phi_S + \sin \phi_S) - v_T (\cos \phi_T + \sin \phi_T)}. \]  

(18)

After some manipulation it can be shown that setting \( \phi_T = \phi_T^+ \) in (18) guarantees that \( t_I > 0 \), under certain conditions. To demonstrate this, one can substitute the solutions for \( \cos \phi_T^+ \) and \( \sin \phi_T^+ \) in (16) into Eq. (18) to give

\[ t_I = \frac{R_0^2}{v_S (x_0 \cos \phi_S + y_0 \sin \phi_S) + v_T \Delta} \]  

(19)

where \( R_0 = (x_0^2 + y_0^2)^{1/2} \) is the initial torpedo-ship range. A solution for which \( t_I > 0 \) occurs if \( \Delta \) is real. Consequently, the torpedo can intercept the target only if

\[ \frac{v_S}{v_T} D_\perp < R_0. \]  

(20)

If \( v_T > v_S \) this condition is automatically satisfied because \( D_\perp \leq R_0 \). It is only in the case where the torpedo travels more slowly than the ship that the possibility of an intercept is dependent on the initial geometry. Equation (20) can be rearranged to yield the minimum torpedo speed necessary to achieve intercept for given initial conditions. In this study the torpedo always travels faster than the ship and one does not have to test (20).
The intercept course may be derived in an alternative way, by consideration of the triangle that is formed by the ship course, the torpedo’s intercept course, and the line connecting the initial torpedo-ship positions. First, calculate the bearing of the ship relative to the torpedo, $\psi$, from

$$\psi = \arctan \left( -y_0, -x_0 \right)$$

(21)

using the four-quadrant arctangent function. Then the torpedo’s intercept course, $\phi_T$, is given by

$$\phi_T = \sin^{-1} \left[ \frac{v_S}{v_T} \sin(\phi_S - \psi) \right] + \psi$$

(22)

and the time until intercept is

$$t_I = \frac{R_0}{v_T \cos(\phi_T - \psi) - v_S \cos(\phi_S - \psi)}.$$  

(23)

### 4.4 Sonar

In this study we consider the torpedo to have passive sonar only. The torpedo sonar array consists of a $6 \times 6$ element rectangular array spaced for a centre frequency of 25 kHz giving an element separation of 0.03 m in the horizontal and vertical directions. The sonar processor forms seven horizontal beams with an inter-beam separation of $7^\circ$ from the array. The array is mounted on the nose of the torpedo, three metres in front of its geometric centre.

The torpedo sonar is assumed to be a broadband sonar system with an output bandwidth of 2 kHz, centred on 25 kHz. The sonar is coupled to a square-law detector, in which the signal from the array is squared and time averaged. However, to simplify the model the detection process is treated deterministically using the sonar equation, that is, we ignore statistical fluctuations from the detector output [5]. Hence, false alarms arising from noise fluctuations are not considered in this model.

#### 4.4.1 The passive sonar equation for multiple sources

The acoustic intensity incident at the sonar array due to a single source $n$ can be expressed as

$$I_n = 10^{\left( S_{L_n} - T_{L_n} + B_{R_n} \right)/10}$$

(24)
where $SL_n$ is the source level, $TL_n$ is the transmission loss for this source and $BR_n$ is the beam pattern response in the direction of the source. $BR_n$ is assumed to be normalized so that it is zero along the beam’s Main Response Axis (MRA) (the normalization factor is the array gain which is common to all beams and is included later).

The signal excess $SE$ on each beam due to all sources is then given by

$$SE = 10 \log_{10} \left( \sum_{n=0}^{N} I_n \right) - NL + AG - DT. \quad (25)$$

where $NL$ is the noise level, $AG$ is the array gain (for a plane wave signal against the noise background) and $DT$ is the detection threshold. Odin’s passive sonar model also includes a directivity index term, which is used to evaluate the contribution of ambient noise to the total background noise level. The current version of TOAST lacks this facility. Instead, the total noise level on the sonar must be specified in the sonar’s self noise component. It does not matter in this scenario since the torpedo sonar is assumed to be flow noise limited.

Note that TOAST takes source-receiver propagation delay into account when calculating received noise levels. It also models source-receiver Doppler shift in its passive sonar model. For this reason, noise level tables in TOAST simulations should be extended in frequency past sonar receiver bandwidths by an amount that allows for Doppler shifts in the received noise.

All quantities in and (24) and (25) are broadband intensities integrated over the torpedo’s sonar band. A valid beam detection is deemed to occur when $SE > 0$ with probability one. If $SE \leq 0$ then no detection occurs. In general, the detection threshold depends on the details of the detector electronics and signal processing.

Finally, the sonar is assumed have an automatic gain control which keeps the beam outputs within the dynamic range of the digital signal processor at all times.

### 4.4.2 Detection threshold

The torpedo sonar processor is modelled as a broadband power detector for unknown steady signals in a background of Gaussian noise. In this situation the broadband detection threshold is given by ([6], p417)

$$DT = 5 \log_{10} \left( \frac{d}{WT} \right) \quad (26)$$

where $d$ is the detection index, $W$ is the sonar bandwidth, and $T$ is the integration time. If the passive sonar detector measures power then the noise follows an exponential distribution.
for a small number of independent samples. An appropriate expression for the detection index for exponential noise statistics (assuming an integration factor of 1) is given by ([7], p21)

\[ d = \left( \frac{\log_e(P_{FA})}{\log_e(P_D)} - 1 \right)^2 \] (27)

where \( P_{FA} \) is the probability of false alarm and \( P_D \) is the probability of detection. In this study we take \( P_{FA} = 10^{-6} \) and \( P_D = 0.95 \) which gives \( d = 7.2 \times 10^4 \). Through (26) we have \( DT = 7.78 \approx 8 \) dB.

We assume that transmission loss is given by the sum of spherical spreading and frequency dependent absorption,

\[ TL = 20 \log_{10} R + \alpha R \] (28)

where \( \alpha \) is the frequency dependent attenuation coefficient. At 25 kHz the Thorp absorption model yields an attenuation of \( \alpha = 5.56 \) dB/km.

When the ship is travelling on a steady course at 15 kn the source level of the ship is about 143 dB when integrated over the bandwidth of the torpedo sonar (Table 11). The torpedo sonar provides an array gain of \( 10 \log_{10} 36 \approx 15 \) dB, and the self noise when the torpedo travels at 30 kn is about 63 dB (Table 6). The signal excess

\[ SE = SL - TL + AG - SN - DT \] (29)

is then just positive (i.e. the ship is just detected by the torpedo) when \( R \approx 3 \) km for \( DT = 8 \) dB. This is a realistic range for a heavyweight torpedo to acquire a 15 kn frigate.

4.4.3 Beam patterns

Taylor shading is used on the array in the horizontal direction and rectangular shading for the vertical response (the same as the AG-3 generic torpedo). The array uses electronically formed beams steered at the angles \( 0^\circ, \pm 7^\circ, \pm 14^\circ \) and \( \pm 21^\circ \) from the main response axis. The array shading is intended to reduce sidelobe levels in the horizontal plane, to make the torpedo less vulnerable to acoustic decoys. The element weights were computed for a Gamma value of 4.0, and are given in Table 3.

In both Odin and TOAST the total beam pattern may be specified as the sum of separate
Table 3: The (non-normalized) horizontal weights for the torpedo sonar array.

<table>
<thead>
<tr>
<th>Element</th>
<th>Weight</th>
<th>Element</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
<td>4</td>
<td>7.2779</td>
</tr>
<tr>
<td>2</td>
<td>4.3067</td>
<td>5</td>
<td>4.3067</td>
</tr>
<tr>
<td>3</td>
<td>7.2779</td>
<td>6</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

horizontal and vertical responses, viz.

\[
BR(\phi, \theta) = HR(\phi) + VR(\theta) - BF(\alpha) \tag{30}
\]

where HR is the horizontal response, VR is the vertical response and BF is the baffling level due to the torpedo body. The angle \( \phi \) is the azimuth relative to the sonar array axes, \( \theta \) is the elevation, and \( \alpha \) is the angle of incidence on the sonar array. When \( \alpha = 0 \) incident radiation is unaffected by the torpedo body, and so \( BF(0) = 0 \), but when \( \alpha = 180^\circ \) sound is incident from the rear and is subject to the full baffling effect.

Note that when the element weights \( w_{mn} \) are separable, that is \( w_{mn} = w_m w_n \), and the array is planar it can be shown that the beam response automatically separates as per (30). Since in the present scenario the element weights are separable the use of (30) does not introduce any approximation into the beam patterns.

4.4.4 Element response

Transducers suitable for operation at torpedo frequencies are usually of Tonpilz design with piezoceramic stacks ([8], p98). The transducers are mounted in a rigid planar baffle to form an array. When mounted in the array each transducer is assumed to have a transmit/receive beam pattern given by

\[
ER(\alpha) = 20 \log_{10} [(1 - a)|\cos(\alpha)| + a] \tag{31}
\]

for \( 0 < a \leq 1 \) where \( \alpha \) is the angle of incidence. When \( a = 1 \) the pattern is omnidirectional, when \( a = 0 \) the pattern is that due to a dipole. The pattern minimum occurs at broadside incidence and is equal to \( 20 \log_{10}(a) \). In this study we assume \( a = 0.1 \) which gives \(-20 \) dB pattern loss at broadside.
<table>
<thead>
<tr>
<th>( \alpha ) (deg)</th>
<th>ER (dB)</th>
<th>( \alpha ) (deg)</th>
<th>ER (dB)</th>
<th>( \alpha ) (deg)</th>
<th>ER (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
<td>63</td>
<td>-5.9</td>
<td>126</td>
<td>-4.0</td>
</tr>
<tr>
<td>9</td>
<td>-0.1</td>
<td>72</td>
<td>-8.4</td>
<td>135</td>
<td>-2.7</td>
</tr>
<tr>
<td>18</td>
<td>-0.4</td>
<td>81</td>
<td>-12.4</td>
<td>144</td>
<td>-1.6</td>
</tr>
<tr>
<td>27</td>
<td>-0.9</td>
<td>90</td>
<td>-20.0</td>
<td>153</td>
<td>-0.9</td>
</tr>
<tr>
<td>36</td>
<td>-1.6</td>
<td>99</td>
<td>-12.4</td>
<td>162</td>
<td>-0.4</td>
</tr>
<tr>
<td>45</td>
<td>-2.7</td>
<td>108</td>
<td>-8.4</td>
<td>171</td>
<td>-0.1</td>
</tr>
<tr>
<td>54</td>
<td>-4.0</td>
<td>117</td>
<td>-5.9</td>
<td>180</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4: Transmit/receive radiation pattern for the generic transducer.

![Figure 2](image)

**Figure 2:** The response of a single transducer element and the response of the forward looking beam.

### 4.4.5 Array baffling

Baffling levels from the class B generic heavyweight were used in the 2007 AG3 study [3]. For this study the baffling levels in Tab. 3 are recommended. The angle in the table is the angle which an incident plane wave makes with the torpedo axis. The baffle is assumed to be a circular disc with an axisymmetric response. Levels in between the tabulated values are to be obtained by linear interpolation.

Baffle levels for torpedo sonars are typically from 30 to 50 dB at 180° incidence. Radiation from the rear enters the sonar array by diffracting around the torpedo body. Hence the baffling level is expected to increase with increasing frequency.

The maximum baffle level of 50 dB may be too high for a 25 kHz array. Future studies may wish to consider the effect of reducing the level to 40 dB. The present level is favourable to
the torpedo as it reduces the chance of reverse steering\(^6\).

<table>
<thead>
<tr>
<th>Angle (deg)</th>
<th>Baffle Level (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>180</td>
<td>50</td>
</tr>
</tbody>
</table>

**Table 5:** Baffle levels as a function of the angle an incident plane wave makes with the torpedo axis.

### 4.5 Self noise

The torpedo sonar is assumed to be flow noise limited with a 7th power dependence on the speed. The noise level on each beam is given by

\[
SN = 30 + 70 \log_{10} \left( \frac{u}{30} \right) \text{ dB re } 1\mu\text{Pa}^2/\text{Hz},
\]

where \(u\) is the weapon speed in knots. Computer simulation models such as Odin and TOAST do not use Eq. (32) directly, but rather interpolate noise levels from a table dependent on speed and frequency. Equation (32) was used to prepare Table 6 which contains the flow noise spectral level for a range of speeds at 25 kHz. The total noise in the beam must be determined by integrating across the full sonar bandwidth. The flow noise spectral level is assumed to be constant over this frequency interval.

<table>
<thead>
<tr>
<th>Speed (m/s)</th>
<th>SN (dB)</th>
<th>Speed (m/s)</th>
<th>SN (dB)</th>
<th>Speed (m/s)</th>
<th>SN (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.0</td>
<td>29.1</td>
<td>17.0</td>
<td>32.9</td>
<td>20.0</td>
<td>37.9</td>
</tr>
<tr>
<td>15.4</td>
<td>30.0</td>
<td>18.0</td>
<td>34.7</td>
<td>20.6</td>
<td>38.7</td>
</tr>
<tr>
<td>16.0</td>
<td>31.1</td>
<td>19.0</td>
<td>36.3</td>
<td>21.0</td>
<td>39.4</td>
</tr>
</tbody>
</table>

**Table 6:** The spectral level of torpedo Self Noise (SN) at 25 kHz (dB re 1\(\mu\)Pa\(^2\)/Hz).

\(^6\)Reverse steering is where the torpedo receives a strong acoustic signal on a side or rear lobe of a beam. The torpedo processor may incorrectly interpret the high level of the beam output to be due to a target in the main lobe of the beam and steer in this direction.
4.6 Torpedo counter-countermeasures

In the passive search mode the torpedo compares sonar beam levels to identify the loudest acoustic source, and then steers in that direction. It is thus vulnerable to acoustic countermeasures which have source levels greater than the target ship.

In the present scenario the torpedo sonar functions as a broadband power detector and cannot discriminate between the signal generated by a countermeasure and the acoustic signature of the ship. This mode of operation is not unrealistic as at torpedo sonar frequencies the noise from the ship is dominated by cavitation and does not contain significant narrow band components. The noise will be modulated at the blade passage frequency [9], but the torpedo’s sonar processor is assumed in this study to be insensitive to this. Hence, in the torpedo’s sonar band both the ship and countermeasure appear as broadband noise.

When the torpedo homes on a decoy and overruns its position there will be a sudden reduction in the sonar beam output level. This is due to the countermeasure moving into the stern arc of the torpedo which is screened by the sonar baffle. This sudden reduction in noise level can be used by the torpedo to detect the presence of the countermeasure.

In the TOAST simulation model this can be done by directly calculating the change in noise level and comparing it with a predetermined threshold. However, in the AG-3 study it was found that the version of Odin in use at DRDC-Atlantic at the time (2006) could not support this. Instead an alternative method of countermeasure detection, compatible with Odin, was devised by B. Vasiliev [10].

In this alternative approach the torpedo is assumed to possess an additional sonar with a single forward looking beam. This sonar has a high output threshold that, if exceeded, indicates the presence of a countermeasure in the beam. This sonar is assumed to be completely blind in the stern arc and is tuned to detect a 145 dB/Hz countermeasure at 100 metres range. It has a centre frequency of 25 kHz, a bandwidth of 2 kHz, and a detection threshold of 75 dB. The self noise of the countermeasure detection sonar is the same as the main torpedo sonar, which was given in Table 6. The beam response is given in Table 7.

At a speed of 30 kn the self noise level is 30 dB/Hz. The signal excess is then \( SE = 145 - 20 \log_{10}(100) - 30 - 75 = 0 \) and so the sonar just detects a 145 dB/Hz source at 100 m range. The maximum source level of the ship in this scenario is 123 dB/Hz, which is achieved when the ship begins its evasive turn. The countermeasure detection sonar will be triggered by the ship at a range of about 8 m from the ship’s radiated noise source (Section 5.2).

\footnote{This sonar system is not physically realizable. It is used solely to enable a direct comparison with the Odin model.}
There is a possibility that the torpedo will falsely identify the ship as a countermeasure before the weapon has a chance to detonate. However, the ship is only very briefly in this high noise state when it initiates the evasive turn on detection of an incoming torpedo. At this time the torpedo is not close enough to the ship for its countermeasure detection logic to be triggered. By the time the torpedo has approached the vessel for an attack, the ship is on a steady course and its radiated noise level has dropped to 109 dB/Hz. At this level the countermeasure detection sonar will be triggered by the ship at a range of about 1.5 m. However, at this range the torpedo is within detonation range of the target point on the hull (Section 4.7.9). We conclude that there is negligible chance of the torpedo’s countermeasure detection logic being falsely triggered by a close approach with the ship.

<table>
<thead>
<tr>
<th>Angle [deg]</th>
<th>HR [dB]</th>
<th>VR [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-180.00</td>
<td>-999</td>
<td>-999</td>
</tr>
<tr>
<td>-90.01</td>
<td>-999</td>
<td>-999</td>
</tr>
<tr>
<td>-90.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>90.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>90.01</td>
<td>-999</td>
<td>-999</td>
</tr>
<tr>
<td>180.00</td>
<td>-999</td>
<td>-999</td>
</tr>
</tbody>
</table>

**Table 7:** The Horizontal Response (HR) and Vertical Response (VR) for the decoy discrimination sonar.

### 4.7 Guidance control

The torpedo is completely autonomous from launch onwards, i.e. there is no guidance wire connecting the torpedo to the firing submarine. The sections below describe the various phases of the torpedo’s guidance and control logic.

#### 4.7.1 Startup

The range of the torpedo from the target at time of launch is uniformly distributed in the interval [5, 7] km. The bearing of the weapon relative to the target at launch is uniformly distributed in the sector [−60, 60] degrees around the broadside of the target ship. The weapon is assumed to have perfect knowledge of the target vessel’s initial position, speed, and course. The intercept fire control course is determined from Eq. (17), and the intercept time from Eq. (18). At $t = 0$ the weapon enters the *StraightRun* phase on the fire control
course at the search speed of 30 kn.

4.7.2 Runout

The weapon travels along the intercept course determined in 4.7.1 at 30 kn. The torpedo continues on this course until $t = 0.75t_I$ at which time it enters the StraightSearch phase.

4.7.3 LinearSearch

In this phase the weapon maintains its current course and reduces speed to search speed (30 kn). The weapon remains in this phase until either a target has been detected or it has spent more than 30 seconds in this phase. If a valid target is detected before the phase has expired the weapon goes into the Verification phase. If the phase duration of 30 is exceeded before a valid target is found the weapon enters the CircleSearch phase.

4.7.4 Verification

The weapon should only enter this phase when one or more detections have been made. The weapon records the beam number with the largest positive signal excess (see Eq. (25)). The torpedo maintains current course and speed for two seconds and then rechecks the sonar beams for targets. If there is another detection on the same beam number a valid target is deemed to have been found and the weapon enters the Homing phase. If there are no detections in this phase, or a detection on a different beam to that on entry, the weapon returns to the StraightSearch phase.

4.7.5 Home

The weapon enters this phase if it believes it has detected a valid target. The weapon checks the sonar beam outputs once every second, and steers in the direction of the beam with the largest positive signal excess. The time at which a contact is made by the sonar is recorded. In the event of a loss of sonar contact the torpedo reduces speed to 30 kn, and holds course for 10 seconds after the time the last contact was made. If contact is not regained within this time the torpedo enters the CircleSearch phase.

If the signal excess on the beam containing the target rises to 10 dB or greater the weapon increases speed to 40 kn. If the signal excess drops below this threshold at any time the speed drops back to the 30 kn search speed.
Listing 1: Countermeasure Detection Logic. Flag = 0 initially.

```
if (Flag = 0) and (DecoyDetected = 1) then
    Flag = 1
else
    if (Flag = 1) and (DecoyDetected = 0) then
        Flag = 0
        goto DecoyOverrun
    end
end
```

The following logic in listing 1 is used to see if the weapon has run past an acoustic countermeasure or decoy. A new simulation always begins with Flag = 0. Effectively, the logic detects the large drop-off in noise level that occurs when the torpedo runs past a loud decoy.

4.7.6 CircleSearch

The torpedo turns at a constant clockwise rate of 10 deg/s and reduces speed to 30 kn. While in this phase the sonar is checked for a new contact every second. If a contact is found the turn is halted immediately and the torpedo enters the Verification phase.

Note that in the Odin SimpleMotion dynamics model the turn rate of the weapon is a step function of time. This means that the turn can be halted immediately upon detection and the torpedo’s course will be the same as that when the target was detected. With other motion models, for example Odin’s ComplexMotion, changes in turn rate occur gradually and thus the beam on which the detection was made may no longer be pointing at the target when the Verification phase is entered.

When more realistic dynamics models are used this may cause the torpedo to reacquire the target on a different beam, which will cause the Verification phase to fail incorrectly. If the torpedo motion model is upgraded to yield more realistic behaviour then this aspect of the scenario logic may require modification.

4.7.7 SnakeSearch

Some experiments with a snake search manoeuvre were undertaken. Define the “snake angle” $\theta_s$ to be half the angular sector swept out by the torpedo’s axis during he snake search, and $R_{\text{max}}$ as the torpedo maximum turn rate. Then define $R_{\text{snake}}$ to be the torpedo turn rate during the snake maneuver.
In this scenario $R_{\text{snake}} = +3.5 \text{ deg/s}$, $R_{\text{max}} = 10 \text{ deg/s}$, and $\theta_s = 45^\circ$. The value of $R_{\text{snake}}$ is equal to half the sonar beam width, this was chosen to keep a target approximately within a beam for the sonar integration time (1 s).

The snake search pattern is defined by the following steps.

1. Set speed to 30 kn.
2. Change course by $\theta_s$ to port and wait for $\theta_s/R_{\text{max}}$ seconds to establish the new course.
3. Set turn rate $R = R_{\text{snake}} \text{ deg/s}$. Maintain for $2\theta_s/|R_{\text{snake}}|$ seconds. During this time the monitor sonar beams for a contact. If a positive signal excess is found go to Verify.
4. Set $R_{\text{snake}} = -R_{\text{snake}}$ (i.e. reverse turn direction), then go to (3).

Some numerical problems were encountered in implementing the $SnakeSearch$ phase in TOAST, probably due to round-off error and/or small inaccuracies in the initial course from the nominal value. If these problems are not corrected the torpedo tends to veer gradually either to port or starboard from the initial course. This problem was simply corrected by hard setting the torpedo’s azimuth to precomputed maximum port/starboard courses at the end of each snake leg. The course corrections required are small and are not believed to unfairly bias the simulation results.

4.7.8 DecoyOverrun

The torpedo decelerates to 30 kn and continues on its current course for 90 seconds, and then enters the $CircleSearch$ phase (or alternatively a $SnakeSearch$ phase).

4.7.9 Hit

A hit is recorded if the torpedo comes within 20 m of the point ($-50, 0, -5$) m in the target ship’s frame of reference ($z = 0$ corresponds to the sea surface).

5 Ship

The target vessel is a generic frigate. It is initially travelling north at 15 kn. It attempts to maintain this speed throughout the initial version of the engagement, however some reduction in speed is inevitable in turns. The vessel carries a generic sonar system which detects the incoming torpedo at a mean range of 3000 m with standard deviation 250 m. Upon detection of the torpedo the ship executes a $+135^\circ$ turn from its current course.
5.1 Dynamics

To model the target vessel motion we use the Odin ComplexMotion model as described in [11]. This model was originally intended for low-speed submarines, but can also be used to model high speed surface ships with an appropriate parameter choice.

The ComplexMotion model allows for the control of a vessel’s course, depth and speed. Responses to course, depth and speed demands are governed by simple differential equations that are integrated during a simulation. The ComplexMotion model assumes that a vessel’s orientation is specified by yaw and pitch, and that the vessel’s course is equal to the yaw angle (i.e. there is zero angle of drift\(^8\)).

The ComplexMotion model does not take vessel roll into account. Therefore, the effect of roll on offboard countermeasure launch angle is ignored. Future studies may wish to model this effect which will require a more sophisticated vessel motion model.

In ComplexMotion one does not specify changes in pitch directly, but rather a change in depth which is converted proportionally to a pitch demand through the relation

\[
\theta_{\text{dem}} = -G_d (z_{\text{dem}} - z), \quad |\theta_{\text{dem}}| \leq \theta_{\text{max}}.
\]  

(33)

The proportionality constant \(G_d\) is called the depth gain, while \(\theta_{\text{max}}\) is the maximum pitch angle that the vessel is capable of. Note the minus sign compared to the Odin documentation. This is because TOAST uses a right-handed coordinate system with the \(z\)-axis pointing upwards, and the pitch angle is defined as the positive rotation angle around the vessel’s \(y\)-axis. Thus, the vessel has a positive pitch angle when pointing nose-down.

The pitch rate is dependent on the speed and is governed by the equation

\[
\dot{\theta} = G_\theta v E
\]  

(34)

where \(G_\theta\) is called the pitch gain. The quantity \(E\) is termed the elevation in the documentation, and is defined by

\[
E = \begin{cases} 
\text{sign} (\theta_{\text{dem}} - \theta), & G_E |\theta_{\text{dem}} - \theta| > 1, \\
G_E (\theta_{\text{dem}} - \theta), & G_E |\theta_{\text{dem}} - \theta| \leq 1,
\end{cases}
\]  

(35)

where \(G_E\) is known as the elevation gain.

\(^8\)The drift is the angle between a vessel’s velocity vector and its longitudinal axis.
The vessel’s yaw angle $\phi$ is governed by the equation

$$\ddot{\phi} + \frac{1}{\tau_H} \left[ \dot{\phi} - \Omega v H(v, \phi) \right] = 0$$

(36)

where $\tau_H$ is a characteristic time for changes in the vessel’s yaw rate, $v$ is the speed of the vessel, and $\Omega$ is the maximum turn rate of the ship in radians per metre. It is related to the tactical diameter $D$ by

$$\Omega = \frac{2}{D}$$

(37)

since in turning through $2\pi$ radians the vessel covers a distance of $\pi D$ metres. The function $H$ is known as the Helm function and is defined by

$$H(v, \phi) = \gamma \min \left[ \left( \frac{v_{\text{crit}}}{v} \right)^2, 1 \right] f(\phi).$$

(38)

Here, $\gamma$ is a constant related to the rudder angle used. Maximum rudder implies $\gamma = 1$, while $\gamma < 1$ is used to model smaller rudder deflections. For speeds greater than the critical speed $v_{\text{crit}}$ the vessel’s turn rate is degraded. $v_{\text{crit}}$ is related to the Odin parameter $H_c$, known as the helm limit, by $v_{\text{crit}} = \sqrt{H_c}$.

The function $f$ determines the fraction of turn rate required to achieve the demanded course $\phi_{\text{dem}}$. It is defined as

$$f(\phi) = \begin{cases} 
\text{sgn}(\frac{\phi_{\text{err}}}{\phi_{\text{crit}}}) & |\phi_{\text{err}}| > \phi_{\text{crit}} , \\
\frac{\phi_{\text{err}}}{\phi_{\text{crit}}} & |\phi_{\text{err}}| \leq \phi_{\text{crit}} ,
\end{cases}$$

(39)

where $\phi_{\text{err}} = \phi_{\text{dem}} - \phi$.

Updating the course $\phi$ is problematic because of the existence of a $2\pi$ discontinuity in the angle. This issue has already been discussed in Section 4.1. The solution is to use direction cosines to represent a vehicle’s course rather than angles. The course angle, if needed, can always be extracted from the direction cosines. The course error $\phi_{\text{err}}$ should be calculated from the direction cosines using Eq. 9. In the special case where the vessel is ordered to make a $180^\circ$ course change the turn direction is undefined. In a computational implementation the actual turn direction will be determined by round-off error. It is better to manually specify a turn direction should this situation arise.

The use of a second order equation for the yaw has the effect that a vehicle may undergo oscillations about the desired course, in the event that the equation is under-damped. If the vehicle has a sonar system this may cause an anomalous loss of sonar contact as the beam
axes oscillate - this is particularly the case for torpedoes using the \textit{ComplexMotion} model. The present study avoids this problem by using the \textit{SimpleMotion} model for the torpedo dynamics.

The \textit{ComplexMotion} model uses a lagged speed demand model to control speed changes. The governing equations for the speed are

\begin{align}
\dot{F} &= \frac{1}{\tau_v} \left( v_{\text{dem}}^2 - F \right), \quad (40a) \\
\dot{v} &= \alpha (F - v^2) - \lambda_\phi \dot{\phi}^2 - \lambda_\theta \dot{\theta}^2. \quad (40b)
\end{align}

The first of these equations governs the applied “force” \( F \) on the vessel, which is necessary to achieve the demanded speed \( v_{\text{dem}} \). The second governs changes in the vessel’s speed in accordance with the applied force and fluid drag, which is assumed to have a quadratic dependence on the speed. Finally there are two terms which are intended to account for the loss of speed when the vessel makes changes in pitch \( \theta \) or yaw \( \phi \).

Table 8 below lists the parameters used in TOAST’s \textit{ComplexMotion} model. Table 9 shows the relation of the TOAST script \textit{ComplexMotion} parameters to the Odin script parameters. The Odin parameters are on the right hand side of the table. Finally, \textit{ComplexMotion} parameter values for the generic frigate are given in table 10.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Parameter & Name & Units \\
\hline
\hline
\( v_{\text{crit}} \) & MaxYawRateSpeedLimit & m/s \\
\( \tau_H \) & YawDemandLagTime & s \\
\( \tau_V \) & SpeedDemandLagTime & s \\
\( \gamma \) & RudderFraction & \\
\( \phi_{\text{crit}} \) & CriticalCourseError & rad \\
\( \lambda_\phi \) & YawDragCoeff & m/rad^2 \\
\( \lambda_\theta \) & PitchDragCoeff & m/rad^2 \\
\( \alpha \) & SpeedDragCoeff & 1/m \\
\( D \) & TacticalDiameter & m \\
\( G_d \) & DepthToPitchGain & rad/m \\
\( G_\theta \) & SpeedToPitchRateGain & rad/m \\
\( G_E \) & ElevationGain & 1/deg \\
\hline
\end{tabular}
\caption{Parameters in TOAST’s \textit{ComplexMotion} model and the associated script names.}
\end{table}
Table 9: Table showing how to convert from Odin ComplexMotion parameters to their equivalents in the TOAST version of the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>TacticalDiameter</td>
<td>370</td>
<td>m</td>
</tr>
<tr>
<td>MaxYawRateSpeedLimit</td>
<td>8.12</td>
<td>m/s</td>
</tr>
<tr>
<td>YawDemandLagTime</td>
<td>7</td>
<td>s</td>
</tr>
<tr>
<td>SpeedDemandLagTime</td>
<td>30</td>
<td>s</td>
</tr>
<tr>
<td>YawDragCoefft</td>
<td>0.039</td>
<td>m/deg²</td>
</tr>
<tr>
<td>SpeedDragCoefft</td>
<td>0.01</td>
<td>1/m</td>
</tr>
<tr>
<td>CriticalCourseError</td>
<td>22</td>
<td>deg</td>
</tr>
</tbody>
</table>

Table 10: TOAST ComplexMotion parameter values for the generic frigate.

5.2 Radiated noise

The radiated noise of the target vessel was developed by A. Collier at DRDC-Atlantic. The model is based on the radiated noise equation given on p346 of Ref. [5], but modified to take into account the increase in noise following the inception of propeller cavitation. For a frigate class vessel it is assumed that cavitation inception occurs at 12 kn. The radiated noise at torpedo frequencies is assumed to originate from the ship’s propellers. These are lumped into a single point source located at \((-65, 0, -5)\) relative to the ship’s centre.
The vessel’s source level is defined by

\[
SL = 60 \log_{10}(V') + 9 \log_{10}(T) - 20 \log_{10}(f) + 95 - 0.78 \tag{41}
\]

where \( SL \) is the noise level in \( \text{dB re } 1 \mu \text{Pa}^2/\text{Hz} \), \( T \) is the displacement of the vessel in tons, and \( f \) is the frequency in Hz. The correction term of 0.78 dB is to convert from yards to metres. The adjusted speed \( V' \) is related to the vessel speed in knots, \( V \), by

\[
V' = \begin{cases} 
6, & 0 < V \leq 6 \\
0.125V + 5.25, & 6 < V \leq 10 \\
0.25V + 4, & 10 < V \leq 12 \\
2.67V - 25, & 12 < V \leq 15 \\
V, & 15 < V 
\end{cases} \tag{42}
\]

These levels are qualitatively more descriptive of the propulsion noise of modern frigates. For the engagement model scenario it is assumed that \( T = 4000 \).

The increase in radiated noise in a turn can be treated by making the noise level a function of the vessel’s turn rate. Levels intended to be representative of a frigate were also provided by A. Collier. Target vessel noise levels based on Eq. (41), together with the suggested turn rate corrections, are given in table 11.

<table>
<thead>
<tr>
<th>Speed (m/s)</th>
<th>SL @ 0 deg/s</th>
<th>SL @ 0.25 deg/s</th>
<th>SL @ 2 deg/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.56</td>
<td>96.0</td>
<td>99.0</td>
<td>119.2</td>
</tr>
<tr>
<td>7.00</td>
<td>109.2</td>
<td>112.2</td>
<td>123.4</td>
</tr>
</tbody>
</table>

**Table 11:** The radiated noise levels for the target vessel in dB re \( 1 \mu \text{Pa}^2/\text{Hz} \) as a function of absolute turn rate.

In the first version of the present scenario the vessel makes a 135° degree turn to port following detection of the torpedo. The radiated noise increases as the ship undergoes the turn. The total radiated noise in the torpedo’s sonar band (24–26 kHz) is displayed in Fig. 3. The source level of the decoy in the torpedo band is \( 145 + 10 \log_{10}(2000) = 178 \text{ dB} \). In straight course, at 15 kn, the total radiated ship noise is about 142 dB, which is 36 dB below the level of a single decoy. Although the radiated noise of the vessel rises to a maximum of about 156 dB during the turn, this is still 22 dB below the decoy level.
Figure 3: The total radiated noise of the vessel, integrated over the torpedo’s sonar band, as the vessel undergoes a 135° turn to port. Also shown: speed (m/s) and turn rate (deg/s). Key: radiated noise - △; speed - ○; turn rate - ×.

5.3 Sonar

The ship is equipped with a non-specified generic sonar system. The range at which the sonar detects the incoming torpedo is uniformly distributed in the range 2500 – 3500 metres. The detection range is independent of threat bearing and is referenced to the ship’s geometric centre.

5.4 Evasive manoeuvre

Upon detection of the threat weapon the ship undertakes a turn of 135° from its initial course (which is due north, or along the y-axis) away from the half-plane containing the torpedo. The ship’s demanded speed remains the same (15 kn) throughout the manoeuvre. There is a slight loss of speed in the turn which reduces to about 13 kn. Once the vessel is steady on its final course the speed quickly returns to 15 kn.

5.5 Towed acoustic decoy

In the stage 1 of the study the ship does not deploy a towed decoy. However, future evolutions of the scenario may explore the effect of a towed countermeasures used in conjunction with expendable offboard countermeasures.
The model of towed decoy to be used in these subsequent studies is the same as that used in the AG3 study [2]. The decoy is towed by a cable of scope 1500 ft, or approximately 450 m. The cable is towed at a point 65 metres astern of the ship’s centroid, that is, offset from the centroid by $(-65, 0, 0)$ in the ship system of coordinates.

The decoy is an omnidirectional broad-band source that may be operated in a cycled noise mode. The source level is based on the assumption that the power delivered to the projector is 1 kW, the projector efficiency is 25% and the projector bandwidth is 70 kHz. Then, the source level of the decoy is approximately \[ SL = 10 \log \left( 1.43 \cdot 10^{17} \frac{\mu Pa^2}{W} \right) + 10 \log (250 W) - 10 \log (70 \cdot 10^3 Hz), \]
\[ = 145 \text{ dB re } 1 \mu Pa^2/Hz \text{ @ 1 m}. \]

### 5.5.1 Towbody dynamics

A simple geometrical model of towed decoy motion was first introduced as part of the 2007 AG-3 study [2]. For convenience, the essential details of that model are restated here.

The motion of the towed decoy is treated by assuming the cable it is attached to consists of a number of rigid segments connected by links that behave as universal joints at the positions

\[ \mathbf{x}_k, \quad k = 0 \ldots N, \tag{44} \]

where $\mathbf{x}_0$ is the location of the towpoint. At each time step all positions $\mathbf{x}_k$ are updated starting at the towpoint end of the cable. The motion of the towpoint $\mathbf{x}_0$ is determined by the motion of the ship.

The update of the link positions is separated into two steps. The first step adjusts the horizontal coordinates $(x_k, y_k)$ and the second adjusts the height $z_k$. The link height $z_k$ is determined solely by the vessel’s speed, and may be specified by use of a table relating towbody depth to towing speed.

Let the projected point of link $k$ in the sea surface at time step $j$ be denoted by

\[ \mathbf{X}_k^j = (x_k^j, y_k^j). \tag{45} \]

The updated position $\mathbf{X}_k^{j+1}$ is determined by advancing along the line connecting the points $\mathbf{X}_k^j$ and $\mathbf{X}_{k-1}^{j+1}$ until the distance from the latter point is equal to the horizontal length of the segment.

In steady state conditions a homogeneous tow cable has a linear conformation in which it
may be described by the unique depression angle that the cable makes with the sea surface. In the present model it is assumed that when the vessel is maneuvering all cable elements make the same depression angle $\theta_d$ with the sea surface. The value of $\theta_d$ is assumed to depend solely on vessel speed, and adjusts instantaneously in response to changes in speed. Let $l$ be the length of each cable segment. Then the length of each segment projected in the sea surface, $l_p$, is given by

$$l_p = l \cos \theta_d. \quad (46)$$

The updated horizontal locations are determined by

$$\Delta X = X_{j+1}^k - X_j^{k+1}, \quad (47a)$$

$$\alpha = \frac{l_p}{|\Delta X|}, \quad (47b)$$

$$X_{j+1}^{k+1} = X_j^{k+1} + \alpha \Delta X, \quad (47c)$$

for $k = 0 \ldots N - 1$. The depression angle of the cable, $\theta_d(L, V)$, is an exogenous variable and is assumed to be a function solely of the total cable length $L$ and ship’s speed $V$. It is recalculated at each time step, and is independent of the cable shape.

Finally, the towed body is assumed to be located at the end of the cable. It is stabilized so that it travels with constant pitch and roll, and it is aligned with the final cable segment so that its yaw angle $\phi_{TB}$ is

$$\phi_{TB} = \tan^{-1}\left(\frac{y_{N-1}^j - y_N^j}{x_{N-1}^j - x_N^j}\right). \quad (48)$$

In the TOAST implementation of this model the velocity and angular velocity of the towed body are determined from the change in position and the change in yaw angle between steps. This is necessary since other entities, for example the self noise of a towed array, may be dependent on the speed and yaw rate of the towed body.

This model is entirely geometric - it takes no account of the underlying physics of the cable motion, other than preserving the cable length. Nevertheless, it is a useful approximation to the dynamics of real cables. In the limit as the number of segments becomes large the $x$ and $y$ components of the towed body trajectory precisely follow those of the ship. For smaller numbers of segments differences between the towed body and ship trajectories become apparent.

The towed body motion model depends on user specified values of the cable depression angle.
\( \theta_d(L, V) \), as a function of cable scope \( L \) and vessel speed \( V \). In TOAST this is achieved by linear interpolation from an input table. Instead of specifying the cable depression angle in the table it is more convenient to use the towed body depth \( D = L \sin \theta_d \). The depth of the towed body as a function of the ship’s speed is given in Table 12 [12]. The towed decoy runs at about 43 metres depth at 15 kn, and drops to a minimum of 50 metres depth in the turn using this model.

<table>
<thead>
<tr>
<th>Speed (m/s)</th>
<th>Depression Angle (deg)</th>
<th>Towbody Depth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.8</td>
<td>10.4</td>
<td>82.5</td>
</tr>
<tr>
<td>4.0</td>
<td>9.5</td>
<td>75.5</td>
</tr>
<tr>
<td>4.5</td>
<td>8.7</td>
<td>69.2</td>
</tr>
<tr>
<td>5.0</td>
<td>7.8</td>
<td>62.0</td>
</tr>
<tr>
<td>5.9</td>
<td>6.9</td>
<td>54.9</td>
</tr>
<tr>
<td>7.0</td>
<td>6.0</td>
<td>47.8</td>
</tr>
<tr>
<td>8.3</td>
<td>5.2</td>
<td>41.4</td>
</tr>
<tr>
<td>10.0</td>
<td>4.3</td>
<td>34.3</td>
</tr>
<tr>
<td>12.1</td>
<td>3.5</td>
<td>27.9</td>
</tr>
<tr>
<td>16.0</td>
<td>2.6</td>
<td>20.7</td>
</tr>
</tbody>
</table>

Table 12: Cable depression angle and depth of the towed body as a function of vessel speed.

5.6 Expendable acoustic countermeasures

The ship is equipped with two expendable acoustic countermeasure (XACM) launchers. The port launcher is located at the point \((-20, 5, 10) \) m and the starboard launcher at \((-20, -5, 10) \) m relative to the ship’s geometric center.

Each decoy functions as an omni-directional broad-band jammer with a source level of 145 dB re 1 \( \mu \)Pa \(^2\)/Hz. The source spectral level is constant over the band 20 – 30 kHz which covers the torpedo sonar band. The source has a power source which provides an endurance of 8 minutes. The source transmits continuously during this time interval.

In the first version of the scenario each decoy is launched singly, commencing at the time of torpedo detection and then at 30, 60, and 90 s afterwards (a total of four) alternating port and starboard (the first decoy is launched to starboard, i.e. towards the incoming torpedo). Following launch, the decoy travels at an average horizontal speed of 100 m/s. The acoustic
package of the decoy starts transmitting after a time delay proportional to the launch range plus 10 s after splashdown. That is, if a decoy is launched at time $t$ it begins transmitting at a time $t_T$ given by

$$t_T = \frac{R}{100} + t + 10. \quad (49)$$

where $R$ is the desired deployment range from the launch point.

Let $(x_L, y_L, z_L)$ be the coordinates of the launch point at time $t$, $\phi$ be the launch azimuth angle ($\phi = 0$ is the positive $x$-axis), and $R$ be the deployment range. Then the deployment location at $t_T$ will be given by

$$x_D = x_L + R \cos \phi, \quad (50a)$$
$$y_D = y_L + R \sin \phi, \quad (50b)$$
$$z_D = -15. \quad (50c)$$

The current launch model does not consider the effects of velocity addition from the moving launcher on the motion of the decoy. Given that the ship moves about an order of magnitude slower than the decoy this introduces a systematic error of about 10% into the longitudinal splashdown coordinate. Given the generic and explorative nature of the scenario this error is not of serious concern. Future studies may wish to consider more sophisticated decoy flight models.

### 5.6.1 Effective deployment range

When it is deployed an offboard decoy must be within the torpedo’s beam pattern to be effective (although at short ranges a decoy may be loud enough to be detectable on side lobes). An estimate of maximum deployment range that ensures the decoy is just within the torpedo’s sonar beamwidth can be obtained from geometry and certain simplifying assumptions.

Suppose that at time $t$ the ship is located at the origin and a decoy has been launched at $90^\circ$ to the instantaneous ship heading and has deployed at the location $(L, 0)$ (ignore flight time of the decoy).

Assume that the torpedo is located at $(R \cos \phi, R \sin \phi)$ at this time and that its beam pattern center is directed towards the origin (i.e. towards the ship), and the total beam pattern width is $\beta$. Then it can be shown that the maximum lateral displacement $L$, for which the decoy
is just inside the torpedo beam pattern, is
\[ L = \frac{R \sin(\beta/2)}{\sin(\beta/2 + \phi)}, \quad 0 \leq \phi \leq 90^\circ. \] (51)

The minimum value of \( L \) occurs at \( \phi = 90^\circ - \beta/2 \). At this angle we have
\[ L_{\text{min}} = R \sin(\beta/2). \] (52)

For the threat weapon in the current scenario \( \beta = 42^\circ \). The ship detects the torpedo on average at 3 km range and at worst at 2.5 km. Therefore, we take \( R = 2500 \) which gives \( L_{\text{min}} = 896 \) m as an estimate of the effective deployment range \( R_{\text{dep}} \) for the present scenario, based on the analysis above.

This estimate is overly simplistic as it assumes the torpedo has acquired the ship and is homing on its present location. In actuality, the threat torpedo will run out to its homing enable point, which may be closer to the ship than the 2500 m assumed above. And if the ship turns when the torpedo is detected the torpedo’s beam pattern may not be steered in the direction of the ship when acoustic homing is enabled.

6 Scenario A

6.1 Description

This scenario is an engagement between an acoustic homing torpedo and a frigate with an expendable countermeasure capability. The expendable decoys behave as simple broadband omnidirectional noise sources and the torpedo uses passive acoustic homing only.

The torpedo and ship models in this scenario are based on those used in the 2007 AG-3 study [2, 13], and are described in Sections 4 and 5, respectively. However, the torpedo model used in the present work has been updated to be more representative of the capabilities of modern weapons. In particular, it has greater endurance and acquisition range than in the 2007 study.

At \( t = 0 \) the ship is located at \((0,0)\), the coordinate origin, and is heading north at 15 knots. At \( t = 0 \) the torpedo is launched at a distance of \( 6 \pm 1 \) km from the origin, uniformly randomly distributed in bearing from starboard \( 30^\circ \) to \( 150^\circ \) to the ship’s heading. The torpedo is launched on an intercept course, described in Section 4.3. The torpedo runs out
at its search speed until a time = 66% of the calculated intercept time, then enters the acoustic search mode. The guidance and control logic of the current torpedo is based on that given in Ref. [2] and is detailed in Section 4.7.

The ship detects the torpedo at 3 ± 0.5 km with probability one. It then begins an evasion maneuver consisting of a 135° turn to port, while maintaining propulsion settings (there will be a loss of speed in the turn but once complete the ship returns to 15 kn).

The decoys are launched to ranges from 10 to 1800 m at bearings of ±90° relative to the instantaneous ship heading. They are launched singly, starting at the time when the torpedo is first detected, and then at 30, 60, and 90 seconds subsequently. A total of four decoys are launched, alternating between starboard (−90°) and port (+90°). The first decoy is launched to starboard, i.e. towards the incoming torpedo.

The scenario is run 1000 times, with the torpedo launch range and initial bearing randomized uniformly each time. For each run the detection time, hit or miss, time-to-hit, and number of decoy detection events (instances where the torpedo detects a countermeasure) were recorded. From the ensemble of 1000 runs, the hit probability and time-to-hit distributions were determined.

6.2 Sample engagement

Tracks for a particular realization of this scenario are shown in Fig. 4, for increasing time since the start of the engagement. In this realization the torpedo begins at the location $R = 6$ km and $\phi = 10^\circ$, and the ship detects the torpedo at 3 km. The decoys are deployed to a lateral distance of 2000 m from the ship. Ultimately the countermeasures prove to be ineffective and the torpedo hits the ship at $t = 884.5$ s.

The sequence of events in this particular engagement was studied in detail to check the computer model for possible errors. The analysis is given below.

The torpedo begins at the launch point (5910.8, 1041.9) and travels along the intercept course at 30 kn until $t = 268$ s. At this time it drops out of the Runout phase and enters the LinearSearch phase. At 269 s the torpedo makes its first contact on decoy S1. It goes immediately to the Verify phase, but acquires the decoy on a different beam and so drops back to the LinearSearch phase. It reacquires the decoy at 272 s and reenters the Verify phase. This time the torpedo confirms contact at 274 s on the same beam and enters the Home phase. The contact at 274 s is made on the +21° beam with a signal excess of 20.5 dB.

The decoy is not actually within the main lobe of the beam, rather, it has been acquired on a rear lobe, about 45 dB down from the main lobe level. This is possible since the decoy is a
very strong noise source (145 dB/Hz) and the T-S1 range is only 880 m at \( t = 274 \). At this range the transmission loss is approximately \( TL = TL_{spread} + TL_{atten} = 58.9 + 5.2 = 64.1 \) dB (at 25 kHz). The beam pattern level BL in the direction of the decoy at this time was \(-38.3 \text{ dB}^{9}\) (since the decoys are static the retarded position of the decoy noise source does not need to be calculated). The torpedo is moving at its search speed in this phase of the scenario and so the noise level on the torpedo sonar array was 30 dB/Hz.

The signal excess on the 21° beam due to the single decoy S1 is then expected to be \( SE = SL - TL + AG - DT - NL + BL = 145 - 64.1 + 15 - 8 - 30 - 38.3 = 19.6 \) dB. The signal excess calculated by the simulation model was 20.5 dB, which is 0.9 dB above the expected level.

However, there is another decoy which is transmitting at this stage of the scenario, and the ship is also a constant low level source of noise. The second decoy and the ship are much further away from the torpedo than the first decoy, and so will contribute much less to the received level on this beam. The 0.9 dB of additional noise on the beam is due to the second decoy and the ship. We conclude that TOAST is evaluating acoustic levels correctly.

The torpedo then homes on decoy S1 until \( t = 342 \) when it enters the \textit{DecoyOverrun} phase. At this stage the torpedo has approached to within 100 m of the decoy and triggered the decoy detection logic in Section 4.6. The position of the torpedo at the time the \textit{DecoyOverrun} phase is entered was \((2002.5, 1566.0)\) m.

Ninety seconds after decoy S1 is detected, at \( t = 432 \), the torpedo leaves the \textit{DecoyOverrun} phase and enters the \textit{CircleSearch} phase, as required. It immediately makes contact on the nearby decoy P2, and then homes on that decoy. Decoy P2 is detected at \( t = 527 \) forcing the torpedo to reenter the \textit{DecoyOverrun} phase. Ninety seconds later the torpedo initiates another \textit{CircleSearch} phase. It reacquires decoy P1 (the first decoy released to port) and initiates the \textit{Home} phase at \( t = 620 \). At \( t = 696 \) the torpedo has approached decoy P1 close enough to trigger the decoy detection logic. Shortly after this event all four decoys expire.

When the torpedo exits \textit{DecoyOverrun} and enters the \textit{CircleSearch} phase there is only one noise source remaining in the water - the target ship. The torpedo detects the ship, enters the \textit{Home} phase at \( t = 789 \) and finally hits the target ship 884.5 s after the start of the scenario.

We conclude that the TOAST script based on the scenario description correctly describes the scenario, and that TOAST is correctly executing the script.

\footnote{TOAST has the facility to report beam gain levels in the direction of a particular point using the script function \textit{Gain}, defined for objects of type \textit{Beam}.}
### Figure 4: Positions and tracks of the simulation entities for the stage 1 scenario.

In this realization the torpedo is initiated at $R = 6 \text{ km}$ and $\phi = 10^\circ$. Abbreviations:

- T - Torpedo
- S - Ship
- P1 - port decoy 1
- P2 - port decoy 2
- S1 - starboard decoy 1
- S2 - starboard decoy 2
6.3 Ensemble runs

Certain aspects of the scenario are randomized (the initial position of the torpedo, the sonar detection ranges) and consequently each run may lead to different outcomes. In this study the scenario has a binary result - the ship either survives or is hit by the torpedo. The probability of ship survival $p$ can be estimated through repeated runs of the scenario. The sample estimate of $p$ for $N$ runs of the scenario is

$$\bar{p} = \frac{1}{N} \sum_{n=1}^{N} x_n,$$

where $x_n = 0, 1$ is the outcome of trial $n$. When $N$ is large we have $\bar{p} \sim N(p, \sigma)$ by the central limit theorem, where $\sigma = \sqrt{p(1-p)/N}$. The standard deviation $\sigma$ is maximized when $p = 0.5$ and takes the value $0.5/\sqrt{N}$. When $N = 1000$, and assuming $p = 0.5$, we have $\sigma \approx 0.016$ or about 1.6% error in the estimate for $p$ at the most. This level of accuracy is adequate for the present study and so 1000 runs of each scenario were used to estimate ship survival probability.

Scenario A was run 1000 times for decoy deployment ranges between 10 and 1800 m. Given the cost advantages of hand and pneumatically launched decoys it is important to understand how deployment range influences effectiveness. The results are given in Table 13.

A deployment range ($R_{dep}$) of 10 m is intended to simulate manual- or hand-launched decoys. This approach has the obvious advantage that launchers are not required to be fitted to the vessel. A deployment range of 50 m simulates the effect of pneumatic launching systems, for example, those installed on Royal Navy Type 23 frigates and used in conjunction with the Sonar 2170 torpedo defence system. Pneumatic launchers have the advantage of not requiring storage of explosive material. To project decoys to ranges beyond 100 m rocket propulsion is required. For example, Rafael’s Lescut decoy is available in a form which can be fired from a Mk 36 chaff launcher.

6.4 Torpedo hit probability

The variation in torpedo hit probability with deployment range was substantial. Decoy patterns are relatively ineffective for deployment ranges up to 200 m against the model torpedo (torpedo hit rate 60–70%). Ship survivability improves rapidly as the deployment range increases beyond 200 m and is optimal at 800 m (torpedo hit rate 3.1%). Beyond 800 m the torpedo hit rate begins to gradually increase again, and has reached 30% at 1800 m range.
The guidance control logic in the torpedo is simplistic - it steers towards the loudest acoustic source. If the acoustic level in the forward direction rises above a preset threshold the torpedo assumes the source is a decoy. The torpedo then switches off its acoustic guidance and maintains course for 90 s, at which point it enters an acoustic circle search mode.

The 90 s run after a decoy detection allows the torpedo to clear a field of decoys if they are closely spaced. This gives the torpedo more chance to reacquire the target ship. If the inter-decoy spacing is too great the jamming signal received from a second decoy following a decoy overrun may not be strong enough to mask the ship.

<table>
<thead>
<tr>
<th>XACM deploy range (m)</th>
<th>Torpedo hit %</th>
<th>Mean time-to-hit (mins)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>59.7</td>
<td>10.6</td>
</tr>
<tr>
<td>50</td>
<td>70.0</td>
<td>10.1</td>
</tr>
<tr>
<td>100</td>
<td>70.0</td>
<td>10.1</td>
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<tr>
<td>200</td>
<td>69.9</td>
<td>10.4</td>
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<td>400</td>
<td>51.8</td>
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<td>600</td>
<td>31.5</td>
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<td>800</td>
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<td>16.3</td>
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<td>1000</td>
<td>5.0</td>
<td>11.6</td>
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<td>1200</td>
<td>6.2</td>
<td>12.3</td>
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<tr>
<td>1400</td>
<td>8.0</td>
<td>13.4</td>
</tr>
<tr>
<td>1600</td>
<td>14.9</td>
<td>16.4</td>
</tr>
<tr>
<td>1800</td>
<td>30.9</td>
<td>15.3</td>
</tr>
</tbody>
</table>

**Table 13:** The torpedo hit rate as a function of the XACM (decoy) deployment range. Each decoy was launched at 90 degrees azimuth relative to the ship’s heading. There is a rapid increase in effectiveness of the countermeasures as the deployment range is increased from 600 to 800 m from the ship.

Decoy effectiveness peaks at a deployment range of about 800 m, and declines slowly thereafter. The optimal range found by simulation is actually quite close to the value of 896 m that was suggested in section 5.6.1, using the formula

\[ R_{dep} = R_T \sin \beta / 2 \] (54)

where \( R_{dep} \) is the effective deployment range, \( R_T \) is the range of the torpedo from the ship at deployment time, and \( \beta \) is the torpedo sonar azimuth beam width. That value was obtained from an argument based on deployment time and torpedo beam pattern width, to ensure that the decoy would be within the torpedo’s beam pattern when deployed. The argument
was overly simplistic as it assumed the torpedo’s beam pattern was centered on the ship at the time of decoy deployment, which will often not be the case. In the present scenario, for example, the torpedo may still be in the *Runout* phase when detected by the ship - its beam pattern will be directed towards the expected impact point, not the current location of the ship.

The estimated deployment range given in 5.6.1 may be regarded as a starting point, when no other information is available, which must be refined by simulation. However, the good agreement with the optimal value found by simulation in this case is probably accidental. The value of that estimate could be tested further by increasing the torpedo’s beam width to see if the revised estimate matched the optimal range determined by simulation. This, naturally, would entail considerable computation. Ensemble runs over the feasible range of decoy placement locations for each new beam pattern would need to be generated. The torpedo model itself would need only minor modification - an increase in the number of horizontal beams to allow the beam pattern window to cover azimuths greater than ±21°. Such an analysis was not the primary focus of the present study and has not been attempted here.
6.5 Outcome maps, time-to-hit statistics

The effectiveness of the torpedo in each scenario depends strongly on the torpedo’s initial bearing relative to the ship. This aspect is illustrated in Figs. 6–9 in which scenario outcome is mapped as a function of the torpedo’s initial position. Each figure displays a map of hit-miss outcome for three decoy deployment ranges, as well as a histogram of the time-to-hit (for those simulations which resulted in a hit for the torpedo). The torpedo hit rate and decoy deployment range are shown in each plot. Torpedo hit is indicated by a blue x while as miss is shown by a red o.

Outcomes of short-range (≤ 100 m) deployments are shown in Fig. 6. The torpedo is quite effective in hitting the ship in these cases with hit rates of 60–70%. The outcome maps show a strong azimuth dependence of the torpedo success. The torpedo is most effective when launched in an arc centered slightly ahead of broadside. Torpedo effectiveness peaks at 70% hit rate for decoy deployments in the ranges 50–200 m. Torpedo failures are concentrated for launch positions in the stern arc of the ship. The mean time-to-hit in all of these cases is about 10 mins.

In Fig. 7 the torpedo hit rate has begun to sharply decline for a deployment range of 400 m, with the torpedo losing its effectiveness at the edges of the launch arc. The mean time-to-hit is approximately unchanged, but one can see the development of a bimodality in the behaviour of the torpedo in the second peak in the histogram at about 22 mins. At 600 m range the hit rate has declined to 31.5%. Launch positions abeam of the ship are now contributing the bulk of the torpedo successes.

The torpedo experiences its minimum success rate of 3.1% for a deployment range of 800 m, which is shown in the first plot in Fig. 8. Increases in decoy deployment range from this point on result in steadily increasing success rates for the torpedo, as shown in this figure and also in Fig. 9.

At the longest deployment range considered (1800 m, Fig. 9) the success rate has climbed to 30.9%. In contrast to the earlier scenarios, success is regained near the top of the arc of initial positions. It is likely to be because the decoys are deployed too far from the ship to enter the torpedo’s acoustic window. At these launch angles, when the ship has completed its evasive turn the torpedo will be almost directly astern. This is an ideal location from which to attack, avoiding decoys which have been placed far from the ship.

For attack angles to beam and behind the ship decoys may still be effective even at these long deployment ranges. Following the evasive maneuver the torpedo must turn to acquire the target. In doing so its sonar will sweep through a considerable sector giving it more chance of acquiring a decoy.
Figure 6: Torpedo success versus initial position maps and time-to-hit histograms for scenario A, for XACM (decoy) deployment ranges of 10, 50 and 100 m. Key: x – hit; o – miss.
Figure 7: As Fig. 6, for deployment ranges of 200, 400 and 600 m.
Figure 8: As Fig. 6, for deployment ranges of 800, 1000 and 1200 m.
Figure 9: As Fig. 6, for deployment ranges of 1400, 1600 and 1800 m.
7 Scenario B: effect of reduced alert time

In the original scenario the ship detects the torpedo at 3±0.5 km range. The ship begins its evasive maneuver immediately following detection - a 135° turn to port, followed by decoy deployment. Simulation results have shown that decoy deployment geometry is a crucial factor in determining the scenario outcome.

Another aspect that has not been explored so far is the effect of larger variations in the ship’s alert time, after which the evasion tactic of maneuver and decoy deployment is initiated. In a real engagement it is likely that the range at which the ship acquires the torpedo will be more variable than presently assumed.

To explore this issue the basic scenario was re-executed but with the detection range for the ship against the torpedo set to 1.5 ± 0.25 km. At this reduced range Eq. (52) indicates a maximum effective decoy deployment range of 538 m. Results are given in Table 14.

In this case the torpedo success rate has a bimodal dependence on the deployment range. There is a valley in the torpedo success between 200 and 400 m, followed by an increase, and then a dip at 800 m. Thereafter the torpedo hit rate climbs steadily to over 90% at the longest deployment range considered. As expected there are complexities in the interaction of the torpedo and the decoy deployment pattern that cannot be captured in the simple relation of Eq. (52). On the other hand, the estimate is about midway between the two minima in the torpedo hit rate curve so it serves as a useful starting point for further refinement.

Scenario B indicates that decoys can still be effective even if the torpedo is detected at short ranges from the ship. Maximum effectiveness is attained for a deployment range of 800 m, the same as Scenario A, with a torpedo hit rate of 4.7% (cf. 3.1% in Section 6.4).

8 Scenario C: alternative torpedo counter-countermeasures

Certain choices of the decoy deployment range have caused the torpedo to perform poorly in Scenarios A and B.

Repeated observations of single scenarios revealed a possible weakness in the guidance control logic for the torpedo. After detecting a decoy, the torpedo maintains its course for a preset amount of time (90 s in this scenario) and then enters the CircleSearch mode in an attempt to reacquire the target vessel. Quite frequently the torpedo reacquires the decoy it just passed over in the circle search and reattacks it. This wastes fuel and allows the ship more time to escape.

This guidance control logic may be quite effective against a towed decoy, but clearly can be

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### Table 14: Scenario B
As for Table 13 except the ship now detects the torpedo at reduced ranges in the band $1.5 \pm 0.25$ km, instead of $3 \pm 0.5$ km.

<table>
<thead>
<tr>
<th>XACM deploy range (m)</th>
<th>Torpedo hit %</th>
<th>Mean time-to-hit (mins)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>22.1</td>
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<td>15.7</td>
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<td>12.7</td>
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<td>1800</td>
<td>87.5</td>
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**Figure 10:** The torpedo hit rate (\(--\triangle--\)) and the mean time-to-hit (\(--o--\)) as a function of XACM (decoy) deployment range for Scenario B.

ineffective against multiple static decoys. One potential strategy to deal with static decoys is to store locations of decoys once they are encountered to avoid a later reattack. This capability is in principle within the capability of modern torpedoes, which are equipped with computers and inertial guidance systems. A problem with this approach is that it is potentially less effective against towed decoys.
Another approach to avoid reattack on a static decoy is not to enter a circle search after a decoy encounter, but rather to use a search pattern that avoids the decoy location. A “snake” search pattern, in which the torpedo travels along a sinusoidal path that effectively widens its sonar beam pattern width, satisfies this requirement.

<table>
<thead>
<tr>
<th>XACM deploy range (m)</th>
<th>Torpedo hit %</th>
<th>Mean time-to-hit (mins)</th>
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<td>15.1</td>
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Table 15: Scenario C: As for Scenario A except the torpedo now uses the Snake-Search phase following DecoyOverrun. The mean detection range of the ship on the torpedo is again 3 km.

Here we consider the effect of a snake search pattern which is used instead of the circle search after the torpedo drops out of the DecoyOverrun phase (the snake search pattern was defined in section 4.7.7). The results are given below in Table 15.

Comparing these results with those in Fig. 5 shows that the overall performance of the torpedo is not significantly different with the alternative guidance logic. The optimal deployment range for the ship has increased to 1200 m, for which the torpedo hit rate is 1.6%. We conclude that a snake-search manoeuvre following a decoy encounter is not an effective counter-countermeasure for the torpedo.
9 Summary and conclusions

We have explored the effect of expendable acoustic countermeasures on a passive acoustic homing torpedo. The target ship has dynamic and radiated noise characteristics representative of a frigate sized vessel. The torpedo is a generic electrically propelled heavyweight, with a passive sonar operating at 25 kHz. The torpedo model was based on that used in a previous engagement model study [2]. The previous study was intended to cross-validate a number of engagement modeling tools, rather than explore countermeasure effectiveness.

The present study, in contrast, is intended to serve as a baseline for a program of collaborative research regarding expendable acoustic countermeasures. The torpedo model has been made generic to facilitate more open discussion of results, and has also been enhanced to be more representative of the capabilities of modern electrically propelled torpedoes. Resources permitting, future work is expected to focus on more realistic (possibly classified) torpedo and decoy models.

The generic torpedo in the present study forms seven beams with an inter-beam separation of $7^\circ$. It steers towards the loudest noise source by intensity comparison of the beam output levels. In simulations conducted without the use of decoys (i.e. the ship uses evasive manoeuvre only) the torpedo hits the ship with a probability close to one.

The variation in effectiveness of expendable countermeasures with deployment range from the ship was considered, in three variants of the basic scenario.

Figure 11: The torpedo hit rate (---△--) and the mean time-to-hit (–o–) as a function of XACM (decoy) deployment range for Scenario C.
Scenario A. The ship detects the torpedo at a randomly determined range uniformly distributed between $3 \pm 0.5$ km. At the time of detection the ship begins its evasive manoeuvre (a $135^\circ$ turn to port) and begins decoy deployment. The torpedo hit rate against the ship varied from 3\% (deployment range = 800 m) to 70\% (deployment range 50–200 m).

Scenario B. As for Scenario A except the ship detects the torpedo at a randomly determined range uniformly distributed between $1.5 \pm 0.25$ km. The hit rate vs. deployment range curve was bimodal with minima of 7\% at 200 m and 5\% at 800 m. Despite the reduced alert time the ship survival rate using the 800 m decoy deployment range was almost as good as for Scenario A.

Scenario C. As for Scenario A except the torpedo uses a snake search pattern following a decoy detection. The aim of the alternative search procedure was to improve torpedo effectiveness by avoiding reattack on a decoy. However, this strategy was not successful and the torpedo did not perform any better than in Scenario A. This illustrates that expendable decoys are more robust to variations in torpedo search protocol than towed countermeasures.

These initial results suggest that expendable countermeasures can be highly effective against passive acoustic homing torpedoes. Surprisingly, early decoy deployment was not a decisive factor in ship survival. Deployment pattern was the most important factor in determining the effectiveness of expendable acoustic decoys.
References


[10] B. Vasiliev. E-mail, 10 January 2006.


**Initial Study on Expendable Acoustic Countermeasures for Torpedo Defence**

**ABSTRACT**
An initial study into the effectiveness of expendable acoustic countermeasures on passive acoustic homing torpedoes has been conducted. The scenario has been kept generic to enable open discussion of results. Future evolutions of the study will consider more realistic, and possibly classified, torpedo and decoy models. The results indicate that expendable countermeasures can be effective against passive homing torpedoes if the deployment range from the ship is selected appropriately. Surprisingly, early decoy deployment may not be necessary to ensure ship survival. Deployment pattern was more important in determining scenario outcome than deployment time.
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